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Heat and mass transfer on a rough surface with gas blowing at the wall

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Abstract—A theoretical description of the friction and heat transfer coefficients in a turbulent boundary layer on a rough permeable wall with arbitrary rough elements ($K/\delta = 0.06$, $h/K = 1$–4) is presented.

A THEORETICAL description of the processes of momentum, mass and energy transfer to the wall in flow past rough surfaces encounters certain difficulties attributable to the boundary layer separation at each roughness element and to the diversity of rough surfaces. A characteristic feature of the separation of flow is the formation of closed regions behind each obstruction and accumulation of the retarded liquid in them, which is periodically or continually ejected from the separation zone in the direction normal to the main stream [1–3].

The boundary layer separation zones have not, as yet, been open to theoretical analysis even for the simplest cases of turbulent flow over two-dimensional roughness. Therefore, theoretical consideration of a turbulent boundary layer on a rough surface suggests a particular schematic representation of the flow and of the rough surface itself which would allow one to employ momentum, mass and energy differential equations and to impose boundary conditions on the rough wall.

The majority of researchers have no doubts that the variation in the behaviour of flow past rough surfaces is associated with just the specific mass transfer originating behind each roughness element. However, it was not clear in which way this effect on friction and heat transfer could be quantitatively taken into account.

The information available in literature on the visualization of flow past various rough surfaces [3, 4] makes it possible to schematize the flow and to propose a model which is susceptible to theoretical description.

The patterns of flow past rough surfaces considered in refs. [3, 4] and variation in the dimensionless frequency of the ejections of liquid from stalling zones are presented in Figs. 1 and 2, respectively. Analysis of the test data shows that as soon as the roughness begins to influence the drag, Re_{cr1}, the visible inflow and subsequent outflow (ejection) of fluid from the stalling zones behind each roughness element appear. The time-averaged quantity of the fluid participating in such mass transfer increases with the Reynolds number (Re_K) and reaches a certain constant value (at Re_{cr2}) which remains invariable with a further increase in Re_K (Fig. 2). There exists a steady vortex in the cavity. The fluid flows into this cavity ahead of the roughness element and simultaneously flows out from behind the element at an angle close to 90°. These test data allow one, on the one hand, to associate the increase in drag and heat transfer on the rough surfaces only with the arising mass transfer rate behind the roughness elements and, on the other hand, to identify the originating mass transfer with the simultaneous alternating blowing and suction of gas at the wall.

Correspondingly, the rough surface can be modelled by a smooth surface on which the blowing and subsequent suction of gas of the same intensity is made in the region of each roughness element. According to this model, the effect of the roughness elements on friction and heat transfer manifests itself solely through the magnitude of the mass transfer rate. The beginning of the effect on the friction coefficient is determined by the onset of this mass transfer and the end is determined by the constancy of the mass transfer rate parameter.

The elimination of the stalling zones from consideration should not, presumably, cast great doubts on the calculation of drag in as much as it is well known that, in a hydraulically smooth regime of flow past rough surfaces, the boundary of the stalling zones can be substituted by a smooth surface. However, this is not evident for heat transfer, and the answer should be given by experiment.

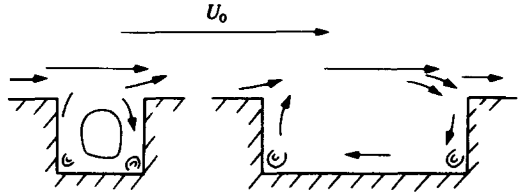

FIG. 1. Pattern of flow past roughness elements.

NOMENCLATURE

- b permeability parameter, $2\bar{j}/C_{fo}$
- $b_{\pm}$ permeability parameter for injection and suction, $2\bar{j}_{\pm}/C_{fo}$
- b_{cr} critical parameter of blowing
- b_T heat penetration parameter, $\bar{j}/St_o$
- C_{fo} friction coefficient under standard flow conditions
- F_{sm} smooth surface area [m^2]
- $\bar{j}$ relative mass velocity in gas blowing
- $\bar{j}_{\pm}$ relative mass velocity in gas blowing and suction
- h spacing between rough elements [m]
- K height of rough elements [m]
- M Mach number
- n frequency of the shedding of vortices [s^{-1}]
- R_o pipe radius [m]
- Re_K, Re^{**} Reynolds numbers, $\rho_o U_o K/\mu_o$, $\rho_o U_o \delta^{**}/\mu_o$
- Re_{cr1}, Re_{cr2} Reynolds numbers corresponding to the beginning and end of roughness effect on drag
- S Strouhal number, nK/U_o
- St_o Stanton number for standard conditions
- U_o flow velocity [m s^{-1}]
- V_{sz} volume of stalling zones [m]
- x longitudinal coordinate [m]
- y transverse coordinate [m]
- Z quantity of rough elements on the area F_{sm}.

Greek symbols

- δ boundary layer thickness [m]
- λ_o longitudinal gradient pressure parameter, $\delta^{**}dU_o/U_o\,dx$
- μ viscosity [kg m^{-1} s^{-1}]
- ρ density [kg m^{-3}]
- $\Psi_{\pm}$, Ψ_r, Ψ_M, Ψ_{λ} relative friction coefficients for equal Reynolds numbers Re^{**}, $C_{f\pm}/C_{fo}$, $C_{f\pm}/C_{fo}$, C_{fM}/C_{fo}, $C_{f\lambda}/C_{fo}$
- ψ, ψ_1, ψ^* relative enthalpies, i_w/i_∞, i_w/i_∞, i^*_w/i_o.

Subscripts

- b blowing
- cr critical
- r rough
- M compressible condition
- sm smooth
- s, T heat transfer condition
- sz stalling zone
- w wall
- o, 0 standard condition or condition beyond boundary layer
- $\pm$ blowing and suction
- λ longitudinal gradient pressure condition.

The laws governing the flow past surfaces with simultaneous blowing and suction of the gas were obtained in refs. [5, 6]. Therefore, it is possible to compare them with test data on the velocity profiles and turbulent friction on rough surfaces which are given in Figs. 3 and 4, respectively.

It is seen from Fig. 3 that there is a satisfactory quantitative agreement between the velocity profiles for the surface with gas blowing and suction and those for the rough surface at identical relative friction coefficients. Figure 4 shows that the distribution of the turbulent friction on the surface with gas blowing

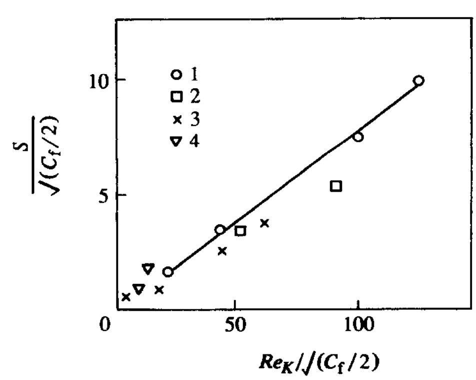

FIG. 2. Variation in the dimensionless frequency of fluid ejections with an increase in the Reynolds number: 1–4, test data [3].

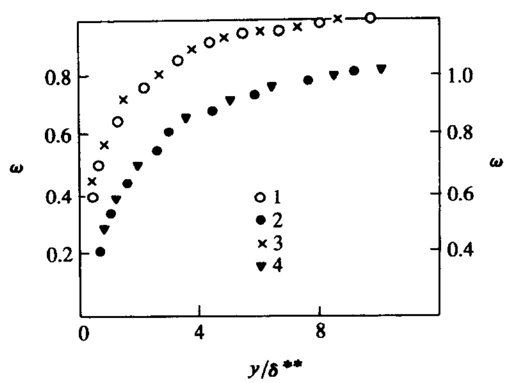

FIG. 3. Comparison of the velocity profiles for flow past surfaces with gas blowing and suction and past rough surfaces: 1, 2, surfaces with blowing and suction at $\Psi_{\pm} = 1.7$ and 2, respectively; 3, 4, rough surfaces at $\Psi_r = 1.64$ and 2, respectively.

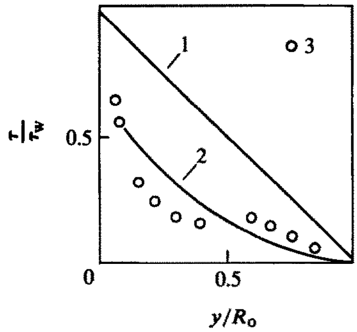

FIG. 4. Comparison of the turbulent friction over the tube cross-section with gas blowing and suction and in a rough tube: 1, calculation from the relation $\tau/\tau_w = (1-y/R_o)$; 2, calculation from the relation $\tau/\tau_w = (1-0.81y/R_o)$ of ref. [5]; 3, test data of ref. [7].

and suction [5] correctly describes the behaviour of turbulent friction on rough surfaces [7]. Moreover, this explains the deviation from the linear law which governs the radial variation of turbulent friction for fully developed flow in rough pipes.

This agreement of the velocity and turbulent friction profiles implies that there exists the similarity between the flows past rough surfaces and those with gas blowing and suction.

Therefore, the relative laws of friction and heat transfer in the case of simultaneous gas blowing and suction [5, 6] are also applicable to permeable rough surfaces

$$\Psi_{\Sigma r} = \Psi_{\pm}\Psi_b = (1+0.0725b_{\pm})^2(1-b/b_{cr0}\Psi_r)^2 \quad (1)$$

$$\Psi_{\Sigma rs} = \Psi_{\pm s}\Psi_{bT} = (1+0.0725Pr^{-0.6}\, b_{T\pm})^2 \times (1-b_T(b_{cr0}\Psi_r\, Pr^{0.6}))^2. \quad (2)$$

The relationship between the parameter $b_{\pm}$ and the geometric characteristics of rough surfaces can be found using the following procedure. It will be assumed that the entire retarded fluid of the stalling zones of volume V_{sz} is periodically ejected into the boundary layer with frequency n. Then the mass transfer rate between the stalling zones and the boundary layer, averaged over the area F_{sm}, can be represented in the form

$$j_{\pm} = \sum_z \rho V'_{sz} n F_{sm}. \quad (3)$$

Taking equation (3) into account, the parameter $b_{\pm}$ can be written as

$$b_{\pm} = 2SZ_{sz}/C_{fo} \quad (4)$$

where

$$S = nK/u_o, \quad Z_{sz} = \sum_z V_{sz}/F_{sm}K.$$

The parameter S is a modification of the Strouhal number, and Z_{sz} is the relative volume of the stalling zones.

With allowance for equation (4), expressions (1) and (2) will take in the form

$$\Psi_{\Sigma r} = [1+0.0725(2SZ_{sz}/C_{fo})]^2(1-b/b_{cr0}\Psi_r)^2 \quad (5)$$

$$\Psi_{\Sigma rs} = [1+0.0725Pr^{-0.6}(2SZ_{sz}/C_{fo})]^2 \times (1-b_T/(b_{cr0}\, Pr^{0.6}\,\Psi_r))^2. \quad (6)$$

Following Fig. 2, a linear relationship will be assumed between $2S/C_{fo}$ and Re_K within the range of Reynolds numbers corresponding to the beginning (Re_{cr1}) and end (Re_{cr2}) of the effect of roughness on drag

$$\frac{S/C_{fo}}{(S/C_{fo})_{cr2}} = \frac{Re_K - Re_{cr1}}{Re_{cr2} - Re_{cr1}}. \quad (7)$$

Substituting equation (7) into equation (5) yields the relative friction coefficient for an impermeable rough surface in the form

$$\Psi_r = \left[1+0.0725Z_{sz}(2S/C_{fo})_{cr2}\frac{Re_K - Re_{cr1}}{Re_{cr2} - Re_{cr1}}\right]^2. \quad (8)$$

The value of $(2S/C_{fo})_{cz2}$ will be obtained from the available test data. Figure 5 presents the outgrowth of a series of Ψ_r reported in various studies for flow over rough surfaces at $Re_K = Re_{cr2}$ and $Z_{sz} = 0.6$–0.7. It is evident from this figure that $\Psi_r = 2$ for these conditions. For $Z_{sz} = 0.65$, equation (8) gives

$$(2S/C_{fo})_{cr2} = 8.8. \quad (9)$$

Assuming that $Re_{cr2}/Re_{cr1} = 14$ on the basis of the test data of ref. [2], taking into account equation (9), equations (5) and (6) for the relative coefficients of friction and heat transfer on rough permeable surfaces can be rewritten as

$$\Psi_{r\Sigma} = \Psi_r\Psi_b = [1+0.049Z_{sz}(Re_K/Re_{cr1}-1)]^2 \times (1-b/b_{cr0}\Psi_r)^2 \quad (10)$$

$$\Psi_{\Sigma rs} = \Psi_{rs}\Psi_b = [1+0.049Z_{sz}\, Pr^{-0.6}(Re_K/Re_{cr1}-1)]^2 \times (1-b_T/(b_{cr0}\, Pr^{0.6}\Psi_r))^2. \quad (11)$$

To determine the Reynolds number for the beginning

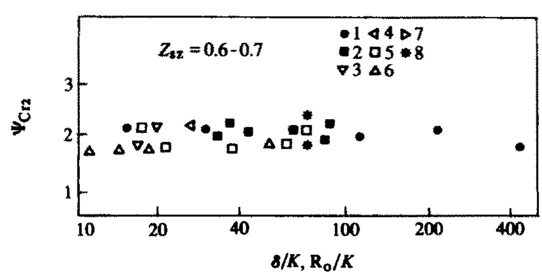

FIG. 5. The relative friction coefficient on rough surfaces at $Re_K = Re_{cr2}$: 1, data of ref. [2]; 2, data of ref. [8]; 3, data of ref. [9]; 4, data of ref. [10]; 5, data of ref. [11]; 6, data of ref. [12]; 7, data of ref. [13]; 8, data of ref. [4].

of the roughness effect on friction, use can be made of the relation from ref. [2]

$$Re_{cr1} = 5/\sqrt{(2/C_{fo})} = 44Re^{**0.125} \qquad (12)$$

whereas the critical parameter of blowing (b_{cr0}) can be predicted using the expression from ref. [14]

$$b_{cr0} = 4(1+5.3Re^{**-0.18}). \qquad (13)$$

The chief merit of equations (10) and (11) is the possibility of calculating the friction and heat transfer coefficients only from the real geometric characteristics of the surface roughness which enter into the parameter Z_{sz}. In particular, the maximum increase of the friction and heat transfer coefficients should be expected on rough surfaces with $Re_K = Re_{cr2}$.

At $Re_K = Re_{cr2}$, it will be obtained for the impermeable rough surface (at $Pr = 0.6$) that

$$\Psi_{rsmax} = (1+0.78Z_{sz})^2. \qquad (14)$$

Equations (10), (11) and (14) are compared with experimental results in Figs. 6–8. In all the cases, a satisfactory agreement can be noted between these equations and the test data. The point to notice is the fact that the critical parameter of blowing b_{cr} is independent of the surface roughness for gas blowing on a rough surface (Fig. 7). This implies that, in the case of critical blowings, there exists such a regime of flow in which no stagnant zones are formed between the roughness element (sharp blowing) and, hence, the velocity profiles for rough and smooth permeable surfaces should coincide. This is confirmed by experimental data [21].

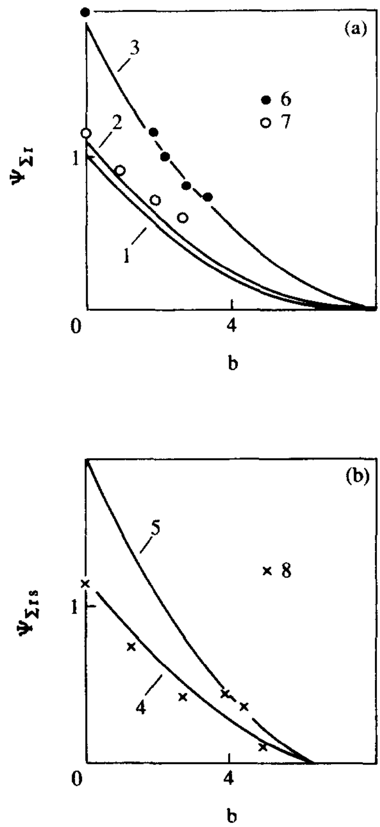

FIG. 7. Comparison of theory and experiment for the relative coefficients of friction (a) and heat transfer (b) on permeable rough surfaces with gas blowing: 1, 2, 3, calculation for $Re_K/Re_{cr1} = 1$, 3 and 14, respectively; 4, 5, calculation from equation (1) for $Pr = 0.6$ and $Re_K/Re_{cr1} = 3$ and 14, respectively. Test data: 6, from ref. [8] for $Re_K/Re_{cr1} = 14$; 7, from ref. [16] for $Re_K/Re_{cr1} = 3$; 8, from ref. [17] for $Re_K/Re_{cr1} = 3$.

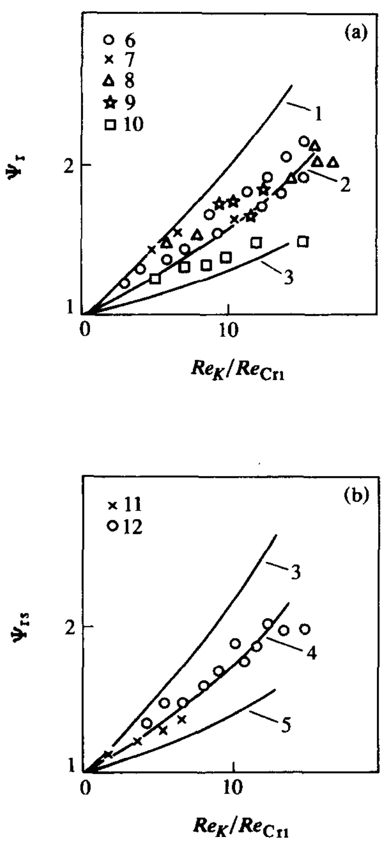

FIG. 6. Comparison of theory and experiment for the relative coefficients of friction (a) and heat transfer (b) on impermeable ($b = 0$) rough surfaces: 1, 2, 3, calculation from equation (10) for $Z_{sz} = 1$, 0.65 and 0.33, respectively; 3, 4, 5, calculation from equation (11) for $Pr = 0.6$ and $Z_{sz} = 1$, 0.65 and 0.33, respectively. Test data for $Z_{sz} = 0.6$–0.7: 6, from ref. [2]; 7, from ref. [3]; 8, from ref. [8]; 9, from ref. [9]; 11, from ref. [8]; 12, from ref. [15]; 10, test data for $Z_{sz} = 0.33$ from ref. [10].

The equations obtained are appropriate for rough surfaces with two- and three-dimensional roughnesses deeply immersed in the boundary layer ($K/\delta \leqslant 0.06$) with the relative pitch $h/K = 1$–4.

Under complex gasdynamic conditions of non-isothermal supersonic gas flow past rough permeable surfaces in the presence of a positive pressure gradient, the following formulae can be suggested for the rela-

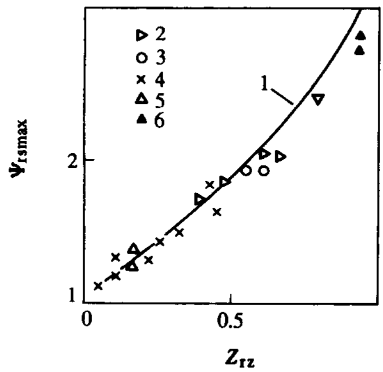

FIG. 8. Maximum increase in the relative heat transfer coefficients on rough impermeable surfaces: 1, calculation from equation (14). Test data: 2, from ref. [12]; 3, from ref. [15]; 4, from ref. [18]; 5, from ref. [19]; 6, from ref. [20].

tive friction and heat transfer coefficients, taking into account the results of ref. [14] and the present author's generalizations for the positive pressure gradient

$$\Psi_{\Sigma r} = \Psi_r \Psi_b \Psi_M \Psi_\lambda = [1+0.049 Z_{sz}(Re_K/Re_{cr1}-1)]^2 \times (1-b/b_{cr0})^2 \left(\frac{\arcsin\sqrt{((\psi^*-1)/\psi^*)}}{\psi^*-1}\right)^2 \times (1-\lambda_o/\lambda_{ocr})^{1.55} \quad (15)$$

$$\Psi_{\Sigma rs} = \Psi_{rs}\Psi_{bT}\Psi_M = [1+0.049 Z_{sz}\,Pr^{-0.6} \times (Re_K/Re_{cr1}-1)]^2 (1-b_T/(b_{sr}\,Pr^{0.6}\,\Psi_r))^2 \times \left(\frac{\arcsin\sqrt{((\psi^*-1)/\psi^*)}}{\psi^*-1}\right)^2. \quad (16)$$

In equations (15) and (16), according to

$$b_{cr} = 4\psi_1^{-0.6}\Psi_M^{1.4}(1+5.3\psi_1^{-0.35}10^{-M/11}\,Re^{**-0.18} \times (1-\lambda_o/\lambda_{ocr}) \quad (17)$$

$$Re_{cr1} = 44 Re^{**0.125}/\sqrt{(\Psi_b\Psi_M\Psi_\lambda)} \quad (18)$$

$$\lambda_{ocr} = 5.3(1-b/b_{cr0})\frac{1.43/\psi^{*0.72}}{1+0.43\psi^{0.9}/\psi^{*0.72}} \quad (19)$$

$$b_{cr0} = 4\psi_1^{-0.6}M^{1.4}(1+5.3\psi_1^{-0.35}10^{-M/11}\,Re^{**-0.18}). \quad (20)$$

The formulae suggested for the relative friction and heat transfer coefficients, along with the integral momentum–energy relations [22], allow one to predict friction and heat transfer for complex gasdynamic conditions of flow over rough permeable surfaces.

REFERENCES

1. R. K. Chang, *Separation of Flow*. Izd. Mir, Moscow (1973).
2. N. Schlichting, *Boundary Layer Theory*. Izd. Nauka, Moscow (1974).
3. H. Townes and R. Sabersky, Experiments on the flow over a rough surface, *Int. J. Heat Mass Transfer* **9**(8), 729–738 (1966).
4. H. Townes, J. L. Gow, R. E. Powe and N. Weber, Turbulent flow in smooth and rough pipes, *J. Basic Engng* **94**(2), 108–119 (1972).
5. A. I. Leontiev, V. G. Puzach, V. P. Komarov, F. K. Yermolayev and N. V. Dubina, Drag of a plate with simultaneous unequal mass blowing and suction, *J. Engng Phys.* **43**(1), 5–9 (1982).
6. A. I. Leontiev, V. G. Puzach, V. P. Komarov, F. K. Yermolayev, N. V. Dubina and V. A. Fadeyev, Heat transfer on the surface of a plate with simultaneous mass blowing and suction, *J. Engng Phys.* **36**(5), 777–778 (1979).
7. R. E. Powe and H. W. Townes, Turbulence structure for fully developed flow in rough pipes, *J. Basic Engng* **95**(2), Ser. D, 162–169 (1973).
8. R. J. Moffat, J. M. Hialzer and W. M. Kays, Experimental study heat transfer behaviour of turbulent boundary layer on rough surface with blowing, *Trans. ASME*, Ser. C, *AIAA J.* **100**(1), 145–155 (1978).
9. P. Antonia and R. Laxton, The response of a turbulent boundary layer to a step change in surface roughness, *J. Fluid Mech.* **48**(4), 721–761 (1971).
10. L. L. Kobzar, Study of hydrodynamic characteristics of a turbulent flow in smooth and rough tubes, Thesis for Candidate's Degree, Obninsk (1973).
11. B. M. Teverovskiy, Towards the problem of the roughness effect on hydraulic drag and convective heat exchange, *Izv. VUZov, Energetika* No. 7, 84–89 (1958).
12. N. M. Galin, Heat exchange in a turbulent liquid flow at rough surfaces, Thesis for Candidate's Degree, Moscow (1966).
13. A. E. Perry, W. H. Shofield and P. N. Joubert, Rough wall turbulent boundary layers, *J. Fluid Mech.* **37**(2), 383–413 (1969).
14. A. I. Leontiev, V. G. Puzach and G. V. Nabatov, Improvement of the limiting relative friction law on a permeable plate with blowing of gas into a compressible turbulent boundary layer, *J. Engng Phys.* **47**(1), 741–748 (1984).
15. V. S. Avduyevskiy, V. N. Kryukov and V. P. Solntsev, Experimental study of the boundary layer structure and heat exchange on a rough surface. In *Study of Heat Exchange in Liquid and Gas Flows*, pp. 55–90. Izd. Mashinostroyeniye, Moscow (1965).
16. J. A. Schetz and B. Nerney, Turbulent boundary layer with injection on surface roughness, *AIAA J.* **15**(9), 90–97 (1977).
17. X. Koulman, M. Pimenta and D. Moffat, Heat transfer to turbulent boundary layer for rough plate with longitudinal gradient U_o, *P*, *T*, *AIAA J.* **16**(1), 99–109 (1978).
18. E. K. Kalinin, G. A. Dreitser and V. A. Kuz'minov, Improving the heat transfer in annular channels, *J. Engng Phys.* **23**(1), 815–818 (1972).
19. A. A. Seleznyov, Roughness effect on heat transfer with a forced air flow in tubes, *Teploenergetika* No. 1, 15–21 (1955).
20. V. K. Migay and I. P. Novozhilov, Heat exchange in the tubes with transverse projections, *Izv. VUZov, Energetika* No. 11, 36–43 (1965).
21. A. I. Alimpiyev and V. I. Alimpiyev, Turbulent boundary layers on a rough permeable plate. In *Turbulent Boundary Layer in Complex Boundary Layers*, pp. 49–59. Izd. ITF SO AN SSSR, Novosibirsk (1977).
22. S. S. Kutateladze and A. I. Leontiev, *Heat Transfer and Friction in a Turbulent Boundary Layer*. Izd. Energiya, Moscow (1972).

TRANSFERT DE CHALEUR ET DE MASSE SUR UNE SURFACE RUGUREUSE AVEC SOUFFLAGE DE GAZ A LA PAROI

Résumé—On présente une description théorique des coefficients de frottement et de transfert thermique dans une couche limite turbulente sur une paroi rugueuse et perméable avec des éléments de rugosité arbitraire (K/δ = 0,06, h/K = 1–4).

WÄRME- UND STOFFÜBERGANG AN EINER RAUHEN OBERFLÄCHE MIT GASEINBLASUNG AN DER WAND

Zusammenfassung—Es wird eine theoretische Beschreibung der Koeffizienten für Impuls- und Wärmeübertragung in einer turbulenten Grenzschicht an einer rauhen, durchlässigen Wand mit beliebigen Rauhigkeitselementen (K/δ = 0,06; h/K = 1–4) vorgestellt.

ТЕПЛОМАССООБМЕН НА ШЕРОХОВАТОЙ ПОВЕРХНОСТИ СО ВДУВОМ ГАЗА НА СТЕНКЕ

Аннотация—Представлено теоретическое описание коэффициентов трения и теплопереноса в турбулентном пограничном слое на шероховатой проницаемой поверхности с произвольными элементами шероховатости (K/δ = 0,06; h/K = 1–4).